Distributed Quantized Transmission and Fusion for Federated Machine Learning

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***Abstract*—Federated machine learning (FL) is a powerful technology which can be implemented to exploit the sheer amount of geographically distributed data for enhanced computation. Exploiting the impending proliferation of wireless devices, in this paper, we incorporate distributed quantized transmissions for reliable connectivity to a remote FL server. We develop a novel theoretical framework for the convergence analysis of the proposed network under joint impact of communication bit error rate (BER), and model quantization, and participation control. We show that the convergence rate of the network is affected by the BER and it can be improved via participation control. Through simulation, we demonstrate that our proposed model can provide the same performance as the conventional FL networks based on point-to-point communication while the energy consumption is divided across the distributed nodes.**

***Index Terms*—Federated Machine Learning, Distributed Systems, Distributed quantization and Fusion, Convergence analysis.**

I. Introduction

DATA centric technologies such as machine learning and artificial intelligence are enabling tools for proposing solutions in a manifold of complex systems [1]. In this regard, the federated machine learning (FL) is an emerging computing technique in which multiple clients take part in training a machine learning model through iteratively sharing their local model updates with an aggregating server instead of sharing their entire dataset. The server then fuses the received updates and feed-back a combined model to the local clients for the next round of training over local datasets. In this context, the *FedAvg* is one of the commonly used combining scheme under which the average of the received model updates are sent back to the local clients [2].

The majority of the related works in the area of FL over wireless links revolves around the coupling between the computation and communication [3]–[9]. Particularly, a resource allocation has been considered in [3] to mitigate the effect of packet error rate on the convergence of the model through power allocation and client selection. In [4], the superposition of the waveforms was exploited to reduce the FL system's latency. The weighted averaging approach was proposed in [5] under which the received model updates from the cellular clients are weighted based on their communication success probability. The energy efficiency of the FL communications was investigated in [6]. Furthermore, some works proposed to employ model quantization to lower the overhead and to improve the latency [7]–[9]. Specifically, model quantization and its impact on the convergence of the FL system was presented in [7]. In [8], different quantization levels for different layers of the neural network layers was proposed. Developing upon the previous works, a joint consideration of the model quantization and transmission outage was taken into account in [9] where authors analyzed the resulting convergence rate.

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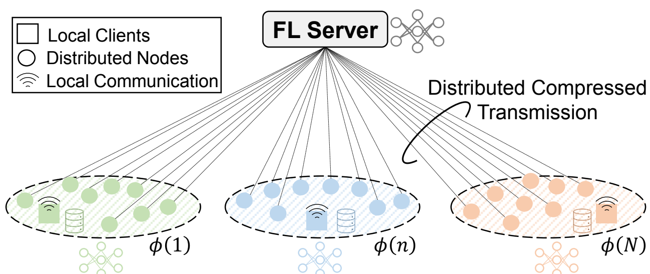

Fig. 1. A conceptual model for the proposed FL through distributed compressed transmission and fusion including N cluster of local clients and distributed nodes with wireless capabilities, denoted $\phi(n)$.

The majority of the related works above focus on the capacity of the wireless channels as one of the main criterion to trigger a client communication, i.e, the aggregation occurs if the wireless link is not in outage. Although effective, this approach overlooks a possible opportunity to exploit the erroneous aggregation links to train the neural network. Particularly, drawing upon the fact that more participation is auspicious in the absence aggregation error [10], it is motivated to determine for what range of bit error rates (BER)s the erroneous aggregated information can be used to expedite the convergence of the FL model.

In this context, the joint impact of BER and model quantization on the convergence of the FL networks has not been investigated yet. Recently, the authors in [11] incorporated the effect of BER on the FL performance under a binary erasure channel where the updates with erroneous communications are excluded from the *FedAvg* at the server. In this paper, we aim to demonstrate that model updates received with communication error can be exploited to improve the convergence of the model training via participation control based on the estimated BER. Furthermore, we introduce a novel FL network model based on distributed quantized transmission and fusion to divide the energy consumption of the network among the several distributed nodes while improving the communication BER [12]–[16]. Our proposed FL network is particularly useful in applications with stringent constraints on power consumption such as satellite communication links. The main contributions of this work are summarized as follows:

- **Network Model:** We introduce a novel FL network based on distributed quantized transmission and fusion to perform FL over unreliable wireless links.
- **Convergence Analysis:** We develop a novel convergence analysis framework and obtain an upper-bound for the convergence rate of the FL system. Different from ralted work [11], our framework includes both of the model quantization and communication BER effects.
- **Verification:** We evaluate the performance of the proposed federated ML network through investigation of its convergence and accuracy for recognition of the hand-written digits using MNIST dataset.

II. System Model

Figure 1 illustrates a conceptual model for our proposed FL network including N clusters of local clients and distributed nodes denoted $\phi(n)$, and the FL server. It is considered that the *FedAvg* is performed at the FL server. At each model aggregation time step, the local client performs the following subsequent steps: (i) first generates the model updates based on its local data and then quantized the model into a lower resolution data. (ii): it transmit towards the neighboring distributed wireless devices to implement distributed quantized transmission towards the FL server. In this paper, we only focus on the uplink phase and we consider perfect communication for the down link [9]. It is considered that all nodes are single antenna and transmit power is fixed. The wireless channel coefficient between the i-th transmitting node inside the n-th cluster and FL server at the t-th time step is denoted $h_{n_i^s}(t)$ where $i \in \{1, ..., |\phi(n)|\}$. For example, $h_{1_2^s}(t_1)$ represents the channel from second distributed node inside the first cluster to the FL server at t_1. The received signals at the receiver is assumed to be perturbed by a zero mean additive white Gaussian noise (AWGN) that its variance is denoted σ^2. In the following, we will first explain the FL aspect of the system and then, we clarify the distributed quantized transmission.

A. Federated Machine Learning

We consider a wireless FL system aimed to fulfill the following computing problem with N local clients:

$$\min_{\boldsymbol{w}} \frac{1}{N} \sum_{n=1}^{N} F_n\left(\boldsymbol{w}\right), \tag{1}$$

where $F_n\left(\cdot\right)$ is the loss function of the n-th client, $\boldsymbol{w} \in \mathbb{R}^Z$ represents the model to be learned with a dimension of Z parameters. Considering the *FedAvg* scheme, the system performs the following key steps:

Coordination: It is assumed that the wireless devices are geographically clustered as distributed nodes neighboring a local client. Prior to transmission towards the distributed nodes, the FL server acquires the channel state information (CSI) of the distributed nodes through dedicated control channels. Based on the CSI, it is then decided what cluster would be triggered for model aggregation.

Local Update: Once participating clusters are determined, each local server trains its model by gradient descent as

$$\boldsymbol{w}_n^t = \bar{\boldsymbol{w}}^{t-1} - \eta \nabla F_n\left(\bar{\boldsymbol{w}}^{t-1}\right), \tag{2}$$

where t is the current time step, η is the learning rate, $\bar{\boldsymbol{w}}^{t-1}$ is the averaged model update from the previous downlink communication time step, and $\boldsymbol{w}_n^t$ is the current local model update. Also, we let g_{nz}^t represent the z-th element of the gradient vector for the n-th local server at the t-th time step, i.e., $\nabla F_n(\bar{\boldsymbol{w}}^t) = [g_{n1}^t, ..., g_{nZ}^t]^T$ where $[\cdot]^T$ signifies transpose.

Gradient Quantization: In this step, the client performs gradient quantization prior to transmission towards its neighboring nodes. We consider stochastic quantization [17] which can be described as the conversion of decimal values into a binary code-word with $p+q+1$ bits, where p bits defines the end points of the quantization intervals denoted by $\Delta_{\min}$ and $\Delta_{\max}$, q bits represents the resolution of quantization between the end points, and 1 bit for the sign of the original decimal value. Mathematically speaking, the z-th gradient parameter of the n-th client after stochastic quantization is given by

$$\mathrm{Q}\left(g_{nz}^t\right) = \begin{cases} \operatorname{sign}\left(g_{nz}^t\right).\mathrm{q}_l & \text{w.p } \frac{\mathrm{q}_l - |g_{nz}^t|}{\mathrm{q}_{l+1}-\mathrm{q}_l} \\ \operatorname{sign}\left(g_{nz}^t\right).\mathrm{q}_{l+1} & \text{w.p } \frac{|g_{nz}^t| - \mathrm{q}_l}{\mathrm{q}_{l+1}-\mathrm{q}_l} \end{cases}, \tag{3}$$

where w.p stands for the 'with probability of', q_l is the l-th quantization interval out of $2^q - 1$ uniformly distributed partitions given by

$$\mathrm{q}_l = \Delta_{\min} + l\left(\frac{\Delta_{\max} - \Delta_{\min}}{2^v - 1}\right), l = 0, ..., 2^q - 1. \tag{4}$$

Uplink Transmission: once local model updates are quantized, the selected clients will be triggered to convey the local updates to FL server through distributed quantized transmission. This phase in the t-th time step of the FL process can be expressed as

$$\bar{\boldsymbol{w}}^t = \bar{\boldsymbol{w}}^{t-1} - \eta \frac{\sum_{n=1}^{N} \alpha_n^t \mathrm{Q}^{-1}\left\{\mathrm{Q}\left[\nabla F_n\left(\bar{\boldsymbol{w}}^{t-1}\right)\right] + \boldsymbol{\chi}_n^t\right\}}{\sum_{n=1}^{N} \alpha_n^t}, \tag{5}$$

where α_n^t is the participation indicator function such that $\alpha_n^t = 1$ if the n-th cluster participate in the uplink transmission and $\alpha_n^t = 0$ otherwise, $\mathrm{Q}\left[\cdot\right]$ represents the quantization process, $\mathrm{Q}^{-1}\left\{\cdot\right\}$ represents the conversion of the digitized data to decimal numbers, and $\boldsymbol{\chi}_n^t \in \{0,1\}^{Z\times(p+q+1)}$ is a binary matrix representing the BER due to the wireless communication. Particularly, each entry of $\boldsymbol{\chi}_n^t$ is a binary vector whose one elements represent erroneous bits.

The communications from the selected clusters are fulfilled through frequency division multiple access (FDMA). Particularly, each local client transmit towards its neighboring distributed nodes at which the observed signal is quantized and then conveyed to the FL server. In the following subsection, we clarify the proposed uplink communication based on distributed quantized transmission and fusion.

In general, the participation parameters can be random variables in that, each local client may not always be available to participate in the aggregation in a predefined and deterministic way. Thus, we assume that each local client will participate with a known a prior probability and we define a super-set $\mathbf{\Lambda}$ encapsulating all possible participation patterns with at least one active cluster:

$$\mathbf{\Lambda} = \Big\{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, ..., \boldsymbol{\alpha}_{2^N-1}, | \, \forall i, j \leq 2^N, \sum_{n=1}^{N} \alpha_{in} \neq 0 \cap \forall i \neq j, \{\boldsymbol{\alpha}_i \wedge \boldsymbol{\alpha}_j\} \neq \boldsymbol{\alpha}_i \Big\}. \tag{6}$$

where $\boldsymbol{\alpha}_i = [\alpha_{i1}, \alpha_{i2}, \cdots, \alpha_{iN}]$ is the i-th possible combination where $\alpha_{in} \in \{0, 1\}$ and $\wedge$ is Boolean intersection. Note that if $\alpha_n^t = 0, \forall n = 1, .., N$, then there will be no communication, and the t-th time step will not count towards the FL process. Therefore, α_n^ts can take on one of the possible patterns in each aggregation time step:

$$\left[\alpha_1^t, \alpha_2^t, \cdots, \alpha_N^t\right] \in \{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, ..., \boldsymbol{\alpha}_{2^N-1}\}.$$

Furthermore, we define the set of active clusters for a given participation pattern $\boldsymbol{\alpha}_i$ to facilitate the convergence analysis. Particularly, we let $\{\boldsymbol{\alpha}_i\}^+$ denote the set of non-zero elements of $\boldsymbol{\alpha}_i$ (the support set) given by

$$\{\boldsymbol{\alpha}_i\}^+ = \{\alpha_{in} | \forall n \in \{1, 2, ..., N\}, \alpha_{in} \neq 0\}. \tag{7}$$

where $|\{\boldsymbol{\alpha}_i\}^+|$ is its cardinality. Similarly, we define the set of corresponding client indexes as

$$\{\boldsymbol{n}_i\}^+ = \{n | \alpha_{in} \neq 0\}. \tag{8}$$

Downlink Transmission: in the last step, the FL server conveys the averaged model update $\bar{\boldsymbol{w}}^t$ down stream to the local clients. Mathematically speaking,

$$\boldsymbol{w}_n^{t-1} = \bar{\boldsymbol{w}}^{t-1}, \forall n = 1, ..., N. \tag{9}$$

In this paper, we assume perfect downlink communication which is common in the FL network literature [9].

B. Distributed quantized transmission and fusion

Letting x_n represent the symbol transmitted by the n-th client, the received signal at the k-th node of the n-th cluster can be written as

$$y_{kn} = \sqrt{\gamma}\mathrm{x}_n h_{nk} + \mathcal{N}_k, \tag{10}$$

where h_{nk} is the complex wireless channel coefficient and $\mathcal{N}_k$ is the additive white Gaussian noise (AWGN) with a variance of σ^2. A PSK modulation is used as $\mathrm{x}_n \in \mathcal{X}$ where $|\mathcal{X}| = 2^M$ and M is the modulation index. We assume that $\mathbb{E}\left[|\mathrm{x}_n|^2\right] = 1$. Each node quantizes the received signal into one-bit per real and imaginary parts and then conveys them to the FL server.

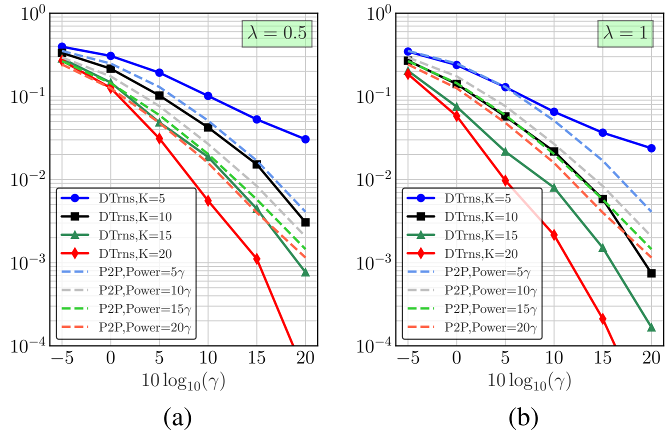

Fig. 2. BER performance of the distributed quantized transmission and fusion technique in comparison to a centralized point to point communication with same total transmit power used, $\lambda = 1$, and 8-PSK modulation.

The received signals from the k-th node of the n-th cluster can be expressed as

$$\hat{y_{kn}} = \text{sign}\{\text{Re}\{y_{kn}\}\} + j\text{sign}\{\text{Im}\{y_{kn}\}\} \tag{11}$$

It is assumed that the available bandwidth is such that the communication error for 1 bit transmission can be neglected [15]. Here, to facilitate the analysis, we express the complex system as a two dimensional real space as

$$\tilde{y}_{kn} = \begin{bmatrix} \text{sign}\{\text{Re}\{y_{kn}\}\} \\ \text{sign}\{\text{Im}\{y_{kn}\}\} \end{bmatrix}, \tilde{\mathrm{x}}_n = \begin{bmatrix} \text{sign}\{\text{Re}\{\mathrm{x}_n\}\} \\ \text{sign}\{\text{Im}\{\mathrm{x}_n\}\} \end{bmatrix}$$

$$\tilde{H}_{nk} = \begin{bmatrix} \text{Re}\{h_{nk}\} & -\text{Im}\{h_{nk}\} \\ \text{Im}\{h_{nk}\} & \text{Re}\{h_{nk}\} \end{bmatrix}, \tilde{\mathcal{N}_k} = \begin{bmatrix} \text{sign}\{\text{Re}\{\mathcal{N}_k\}\} \\ \text{sign}\{\text{Im}\{\mathcal{N}_k\}\} \end{bmatrix},$$

yielding the following real and 2-dimensional formulation of the received signal at the FL server:

$$\tilde{y}_{kn} = \sqrt{\gamma}\tilde{H}_{nk}\tilde{\mathrm{x}}_n + \tilde{\mathcal{N}_k} \tag{12}$$

Adopting the steps taken in [15], the maximum likelihood fusion for detection of the client symbols can be obtained as

$$\hat{\mathrm{x}}_n = \arg\max_{\mathrm{x}' \in \mathcal{X}} \prod_{i=1}^{2} \prod_{k=1}^{K} \Phi(\sqrt{2\gamma}\tilde{H}_{nk}(i)\mathrm{x}'_n), \tag{13}$$

where $\tilde{H}_{nk}(i)$ represents the i-th row of $\tilde{H}_{nk}$ and $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the zero mean and unit-variance normal random variable.

Illustrative example: In Fig. 2, we provide the BER performance of distributed transmission and fusion in comparison to a point-to-point (P2P) communication while maintaining the total transmit power identical between the two systems. As can be seen, as the number distributed nodes increases (K), the distributed technique significantly outperforms the P2P system which is due to its higher diversity gain.

C. Participation control

To enforce participation control, we consider a BER threshold denoted ϵ_{th} such that any cluster with BER higher than ϵ_{th} will not be triggered for the uplink communication. This process is highlighted in **Algorithm 1**.

Algorithm 1: Proposed participation control algorithm

```
Set ε_th and collect the global CSI
for n ∈ {1, ..., N} do
    Find the BER via Monte-Carlo using (13)
    if BER ≤ ε_th then
        α_n^t = 1
    else
        α_n^t = 0
    end
end
```

III. Convergence Analysis

In this section, we develop a theoretical framework to analyze the convergence of the proposed FL network. First clarify some definitions and make some assumption.

A. Preliminaries

1) BER effect: To facilitate the analysis, the erroneous averaged model update due to the BER can be modeled as an additive term to the true update and therefore (5) can be re-written as

$$\bar{\boldsymbol{w}}^t = \bar{\boldsymbol{w}}^{t-1} - \eta \frac{\sum_{n=1}^N \alpha_n^t \mathcal{R}\left[\nabla F_n\left(\bar{\boldsymbol{w}}^{t-1}\right)\right]}{\sum_{n=1}^N \alpha_n^t} - \eta \boldsymbol{\delta}_G^t, \quad (14)$$

where $\mathcal{R}[\cdot] = \mathrm{Q}^{-1}\{\mathrm{Q}[\cdot]\}$ represents the reconstruction from the digital data into decimal values and $\boldsymbol{\delta}_G^t$ is an equivalent random gradient error due to the communication BER.

Lemma 1 Given a participation of $\boldsymbol{\alpha}_i$, $\mathbb{E}_{\boldsymbol{\alpha}_i}\left[\|\boldsymbol{\delta}_G^t\|_2\right]$ for a bit error rate of ϵ can be found as equation (15) in the full version of this paper [18].

2) Gradient quantization effect: Although the model quantization reduces the data overhead [7], it also introduces error affecting the FL performance. The error at the n-th local server can be expressed for each model parameters as

$$e_{nz}^t = g_{nz}^t - \mathrm{Q}\left(g_{nz}^t\right) \forall z = 1, ..., Z. \quad (15)$$

Lemma 2 Under stochastic quantization, the local model updates are unbiasedly quantized as

$$\mathbb{E}\left[\mathrm{Q}\left(\boldsymbol{G}_n^t\right)\right] = \boldsymbol{G}_n^t, \quad (16)$$

where $\boldsymbol{G}_n^t = [g_{n1}^t, ..., g_{nZ}^t]^T$ is the matrix of gradient update and the associated quantization error is bounded as

$$\underbrace{\mathbb{E}\left[\|\boldsymbol{G}_n^t - \mathrm{Q}\left(\boldsymbol{G}_n^t\right)\|_2^2\right]}_{\mathcal{E}_{\mathrm{Q}}^t(n)} \leq \sqrt{\sum_{z=1}^Z \left(\frac{\frac{1}{2}\left(\Delta_{\max} - \Delta_{min}\right)}{2^{p+q+1} - 1}\right)^2}. \quad (17)$$

Proof: Such properties of stochastic quantization have been extensively presented in several works such as [7] and [8].

B. Assumptions

For the computation task, we consider a general smooth non-convex problems under the following assumptions:

Assumption 1: Each local loss function $F_n(\boldsymbol{w})$ is lower bounded as $F_n(\boldsymbol{w}) \geq \underline{F} > -\infty$.

Assumption 2: Each local loss function $F_n(\boldsymbol{w})$s are Lipschitz continuous with constant L, i.e., $\forall \boldsymbol{u}$ and $\boldsymbol{v}$, we have

$$F_n(\boldsymbol{v}) \leq F_n(\boldsymbol{u}) + \left(\boldsymbol{v} - \boldsymbol{u}\right)^T \nabla F_n(\boldsymbol{u}) + 0.5L\|\boldsymbol{v} - \boldsymbol{u}\|_2^2. \quad (18)$$

Assumption 3: The data variance is bounded as

$$\mathbb{E}\|\nabla F(\boldsymbol{u}) - \nabla F_n(\boldsymbol{u})\|_2^2 \leq \mathcal{D}_n^2 \ \forall n \in \{1, ..., N\}. \quad (19)$$

C. Convergence Bound

We build our convergence analysis framework around (18). Due to stochastic quantization and participation control, we focus on the expected values. Using (18) we can rewrite

$$\mathbb{E}\left[F\left(\bar{\boldsymbol{w}}^t\right)\right] \leq \mathbb{E}\left[F\left(\bar{\boldsymbol{w}}^{t-1}\right)\right] + \mathbb{E}\left[\left(\bar{\boldsymbol{w}}^t - \bar{\boldsymbol{w}}^{t-1}\right)^T \nabla F\left(\bar{\boldsymbol{w}}^{t-1}\right)\right] + \frac{L}{2}\mathbb{E}\left[\|\bar{\boldsymbol{w}}^t - \bar{\boldsymbol{w}}^{t-1}\|_2^2\right]. \quad (20)$$

Since the participation parameters create correlated random terms, we use the total expectation to rewrite (20) as

$$\mathbb{E}\left[F\left(\bar{\boldsymbol{w}}^t\right)\right] - \mathbb{E}\left[F\left(\bar{\boldsymbol{w}}^{t-1}\right)\right] \leq \sum_{i=1}^{|\boldsymbol{\Lambda}|} \pi(\boldsymbol{\alpha}_i)\left(\mathbb{E}_{\boldsymbol{\alpha}_i}\left[\left(\bar{\boldsymbol{w}}^t - \bar{\boldsymbol{w}}^{t-1}\right)^T \nabla F\left(\bar{\boldsymbol{w}}^{t-1}\right) \mid \boldsymbol{\alpha}^t = \boldsymbol{\alpha}_i\right] + \frac{L}{2}\mathbb{E}_{\boldsymbol{\alpha}_i}\left[\|\bar{\boldsymbol{w}}^t - \bar{\boldsymbol{w}}^{t-1}\|_2^2 \mid \boldsymbol{\alpha}^t = \boldsymbol{\alpha}_i\right]\right), \quad (21)$$

where $\mathbb{E}[\cdot]$ is a general expectation operator, $\mathbb{E}_{\boldsymbol{\alpha}_i}[\cdot]$ is the expected expectation conditioned on the participation pattern $\boldsymbol{\alpha}_i$ and $\pi(\boldsymbol{\alpha}_i)$ is the probability that $\boldsymbol{\alpha}^t = \boldsymbol{\alpha}_i$ where $\boldsymbol{\alpha}_i \in \boldsymbol{\Lambda}$ as defined in (6). *Note*: From this point on, we discard the condition from conditional expectations for simplicity.

Lemma 3: An upper bound for the gradient of the system loss function can be guaranteed if

$$4\eta L < 1. \quad (22)$$

Proof: *See* Appendix A in the full version of paper [18].

Letting $L = 1/8\eta$ and using ***Lemma 3***, an upper bound for the gradient of the system loss function can be found as

$$\frac{\eta}{4}\mathbb{E}\left[\|\nabla F\left(\bar{\boldsymbol{w}}^{t-1}\right)\|_2^2\right] \leq \mathbb{E}\left[F\left(\bar{\boldsymbol{w}}^{t-1}\right)\right] - \mathbb{E}\left[F\left(\bar{\boldsymbol{w}}^t\right)\right] + \sum_{i=1}^{|\Lambda|}\frac{\pi(\boldsymbol{\alpha}_i)\,\frac{\eta}{2}}{|\left\{\boldsymbol{\alpha}_i\right\}^+|}\sum_{n\in\{\boldsymbol{n}_i\}^+}\mathcal{D}_n^2 + \sum_{i=1}^{|\Lambda|}\frac{\pi(\boldsymbol{\alpha}_i)\,\eta}{2|\left\{\boldsymbol{\alpha}_i\right\}^+|}\sum_{n\in\{\boldsymbol{n}_i\}^+} e_Q^{t-1}(n) + \sum_{i=1}^{|\Lambda|}\frac{\pi(\boldsymbol{\alpha}_i)\,\eta}{4}\mathbb{E}_{\boldsymbol{\alpha}_i}\left[\|\boldsymbol{\delta}_G\|_2^2\right] + \sum_{i=1}^{|\Lambda|}\pi(\boldsymbol{\alpha}_i)\,\eta\mathbb{E}_{\boldsymbol{\alpha}_i}[\|\boldsymbol{\delta}_{\boldsymbol{G}}\|_2]. \quad (23)$$

Averaging both sides of (23) over $\mathcal{T}$ time steps and letting $\mathbb{E}\left[F\left(\bar{\boldsymbol{w}}^T\right)\right] = \underline{\mathcal{F}}$, we obtain

$$\frac{1}{T}\sum_{t=1}^T \mathbb{E}\left[\|\nabla F\left(\bar{\boldsymbol{w}}^{t-1}\right)\|_2^2\right] \leq \quad (24)$$

$$\frac{4\left(\mathbb{E}\left[F\left(\bar{\boldsymbol{w}}^0\right)\right] - \underline{\mathcal{F}}\right)}{\eta} + \sum_{i=1}^{|\Lambda|}\frac{\pi(\boldsymbol{\alpha}_i)}{|\left\{\boldsymbol{\alpha}_i\right\}^+|}\sum_{n\in\{\boldsymbol{n}_i\}^+}\mathcal{D}_n^2 + \sum_{t=1}^T\sum_{i=1}^{|\Lambda|}\frac{2\pi(\boldsymbol{\alpha}_i)}{T|\left\{\boldsymbol{\alpha}_i\right\}^+|}\sum_{n\in\{\boldsymbol{n}_i\}^+} e_Q^{t-1}(n) + \sum_{t=1}^T\sum_{i=1}^{|\Lambda|}\frac{\pi(\boldsymbol{\alpha}_i)}{T}\left(\mathbb{E}_{|\boldsymbol{\alpha}_i}\left[\|\boldsymbol{\delta}_G\|_2^2\right] + 4\mathbb{E}_{|\boldsymbol{\alpha}_i}[\|\boldsymbol{\delta}_{\boldsymbol{G}}\|_2]\right) \quad (25)$$

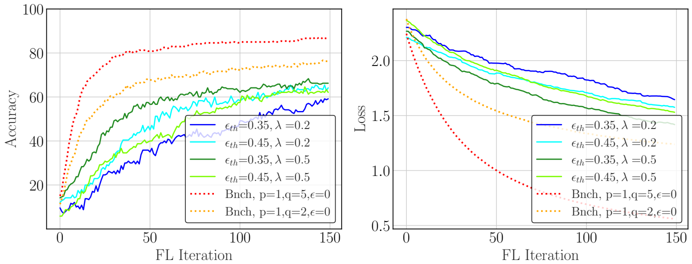

Fig. 3. The joint impact of participation threshold ϵ_{th}, channel gain mean value λ and model quantization quantization $p = 1, q = 2$ on the loss and accuracy of the validation over test dataset. Furthermore, $N = 5$, $M = 8$, and each client data size is 96 samples.

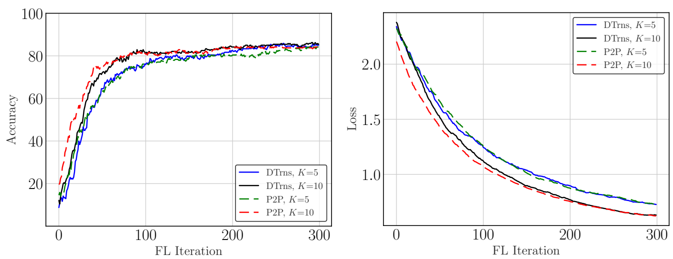

Fig. 4. The performance comparison of the proposed FL network with distributed quantized transmission and fusion versus the FL network based on conventional point to point (P2P) communication. In both systems the model quantization is used with $p = 1$ and $q = 4$, the wireless channel gain is $\lambda = 0.2$, participation threshold is $\epsilon_{th} = 0.35$, $N = 5$, and $M = 8$.

The bound in (25) reveals how quantization error $\mathcal{E}_{\rm Q}^{t-1}(n)$, data variance $\mathcal{D}_n^2$, and BER $\mathcal{E}_G^{\varepsilon}(t)$ affect the convergence of the FL model over iterations.

IV. Experiments & Conclusion

We focus on handwritten digit classification using MNIST data-set which includes 60K training samples and 10K test samples. We consider a simple single layer neural network model and i.i.d local data assumption. The rest of the simulation parameters are given in caption.

Figure 3 respectively illustrates the system's training loss and test accuracy over FL iterations under wireless channels conditions, $\lambda = 0.2$ and $\lambda = 0.5$, and participation control $\epsilon_{th} = 0.35$ and $\epsilon_{th} = 0.45$. Furthermore, as the benchmark scheme, we use a centralized FL with the same computation set up, but without quantization and communication error.

The curves in Fig. 4 demonstrate the joint impact of gradient quantization, BER, and participation control on the accuracy and loss values of the FL network. As can be seen, improving the quantization resolution and reliability translate into better performance. Furthermore, the curves illustrate that the proposed distributed quantized transmission can achieve the performance of the point to point transmission although power consumption is divided among the distributed nodes, avoiding a significant energy consumption demand on a single device. Particularly, the transmit power for the P2P communication is K time bigger than the transmit power of each distributed node in our proposed model.

V. Conclusion

In this paper we proposed a novel FL network based on distributed quantized transmission and studied the convergence of the system under the joint consideration of BER, model quantization, and participation control. We showed that our proposed model can provide the performance of a conventional FL networks with P2P communication, while the energy consumption is divided across the distributed nodes which make our proposed model especially useful for energy-limited applications such as satellite communication networks.

References

[1] M. Vázquez, P. Henarejos, I. Pappalardo, E. Grechi, J. Fort, J. C. Gil, and R. M. Lancellotti, "Machine learning for satellite communications operations," *IEEE Communications Magazine*, vol. 59, no. 2, pp. 22–27, 2021.

[2] H. B. McMahan, E. Moore, D. Ramage, S. Hampson, and B. A. y Arcas, "Communication-efficient learning of deep networks from decentralized data," 2023.

[3] M. Chen, Z. Yang, W. Saad, C. Yin, H. V. Poor, and S. Cui, "A joint learning and communications framework for federated learning over wireless networks," *IEEE Transactions on Wireless Communications*, vol. 20, no. 1, pp. 269–283, 2021.

[4] G. Zhu, Y. Du, D. Gündüz, and K. Huang, "One-bit over-the-air aggregation for communication-efficient federated edge learning: Design and convergence analysis," *IEEE Transactions on Wireless Communications*, vol. 20, no. 3, pp. 2120–2135, 2021.

[5] M. Salehi and E. Hossain, "Federated learning in unreliable and resource-constrained cellular wireless networks," *IEEE Transactions on Communications*, vol. 69, no. 8, pp. 5136–5151, 2021.

[6] Z. Yang, M. Chen, W. Saad, C. S. Hong, and M. Shikh-Bahaei, "Energy efficient federated learning over wireless communication networks," *IEEE Transactions on Wireless Communications*, vol. 20, no. 3, pp. 1935–1949, 2021.

[7] A. Reisizadeh, A. Mokhtari, H. Hassani, A. Jadbabaie, and R. Pedarsani, "Fedpaq: A communication-efficient federated learning method with periodic averaging and quantization," in *Proceedings of the Twenty Third International Conference on Artificial Intelligence and Statistics* (S. Chiappa and R. Calandra, eds.), vol. 108 of *Proceedings of Machine Learning Research*, pp. 2021–2031, PMLR, 26–28 Aug 2020.

[8] S. Zheng, C. Shen, and X. Chen, "Design and analysis of uplink and downlink communications for federated learning," *IEEE Journal on Selected Areas in Communications*, vol. 39, no. 7, pp. 2150–2167, 2021.

[9] Y. Wang, Y. Xu, Q. Shi, and T.-H. Chang, "Quantized federated learning under transmission delay and outage constraints," *IEEE Journal on Selected Areas in Communications*, vol. 40, no. 1, pp. 323–341, 2022.

[10] O. A. Wahab, A. Mourad, H. Otrok, and T. Taleb, "Federated machine learning: Survey, multi-level classification, desirable criteria and future directions in communication and networking systems," *IEEE Communications Surveys & Tutorials*, vol. 23, no. 2, pp. 1342–1397, 2021.

[11] M. Shirvanimoghaddam, A. Salari, Y. Gao, and A. Guha, "Federated learning with erroneous communication links," *IEEE Communications Letters*, vol. 26, no. 6, pp. 1293–1297, 2022.

[12] T.-Y. Wang, Y. Han, P. Varshney, and P.-N. Chen, "Distributed fault-tolerant classification in wireless sensor networks," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 4, pp. 724–734, 2005.

[13] Z. Liu and L. Dai, "A comparative study of downlink MIMO cellular networks with co-located and distributed base-station antennas," *IEEE Transactions on Wireless Communications*, vol. 13, no. 11, pp. 6259–6274, 2014.

[14] J. Choi, D. J. Love, and T. P. Bidigare, "Coded distributed diversity: A novel distributed reception technique for wireless communication systems," *IEEE Transactions on Signal Processing*, vol. 63, no. 5, pp. 1310–1321, 2015.

[15] J. Choi, D. J. Love, D. R. Brown, and M. Boutin, "Quantized distributed reception for mimo wireless systems using spatial multiplexing," *IEEE Transactions on Signal Processing*, vol. 63, no. 13, pp. 3537–3548, 2015.

[16] J. Sun, W. Yi, P. K. Varshney, and L. Kong, "Resource scheduling for multi-target tracking in multi-radar systems with imperfect detection," *IEEE Transactions on Signal Processing*, vol. 70, pp. 3878–3893, 2022.

[17] M. M. Amiri, D. Gunduz, S. R. Kulkarni, and H. V. Poor, "Federated learning with quantized global model updates," 2020.

[18] O. M. Kandelusy, C. G. Brinton, and T. Kim, "Distributed quantized transmission and fusion for federated machine learning," doi = 10.13140/RG.2.2.20497.81762.